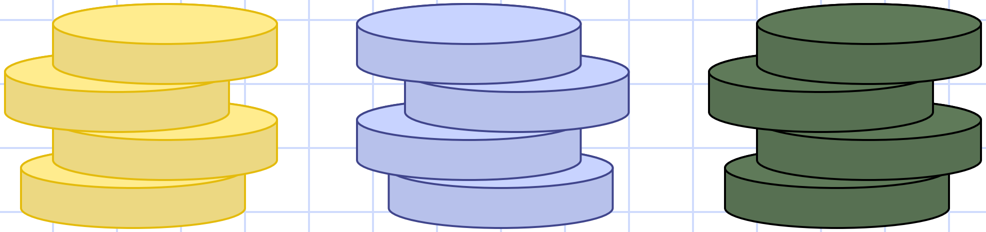
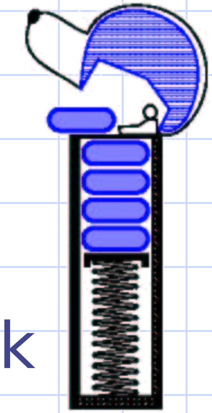
Stacks



Abstract Data Types (ADTs)

- An abstract data type (ADT, for short) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow a last-in first-out scheme
- Think of a spring-loaded plate dispenser
- **Main** stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- **Auxiliary** stack operations:
  - object top(): returns the last inserted element without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored

Stack Interface in Java

- Java interface corresponding to our Stack ADT
- Needs the definition of a class EmptyStackException
- Different from the built-in Java class java.util.Stack

```
public interface Stack<E> {

  public int size();

  public boolean  isEmpty();

  public E top()
    throws EmptyStackException;

  public void push(E e);

  public E pop()
    throws EmptyStackException;
}
```

Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be “thrown” by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- The execution of pop or top on an empty stack throws an `EmptyStackException`

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in the Java Virtual Machine
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

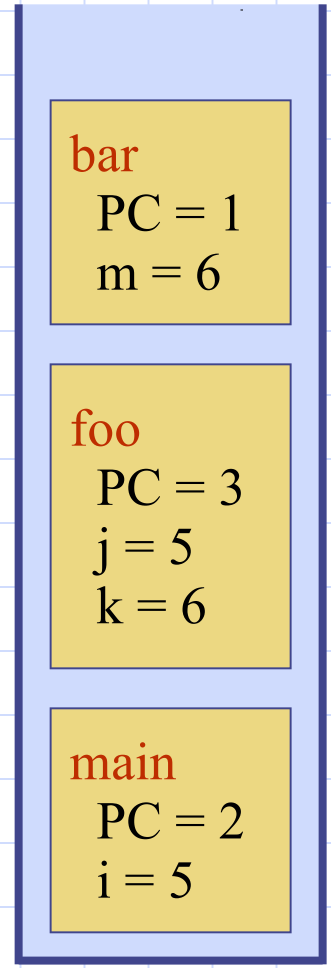
Method Stack in the JVM

- The Java Virtual Machine (JVM) keeps track of the chain of active methods with a stack
- When a method is called, the JVM pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a method ends, its frame is popped from the stack and control is passed to the method on top of the stack
- Allows for recursion

```
main() {
   int i = 5;
   foo(i);
}

foo(int j) {
   int k;
   k = j+1;
   bar(k);
}

bar(int m)
{
   …
}
```

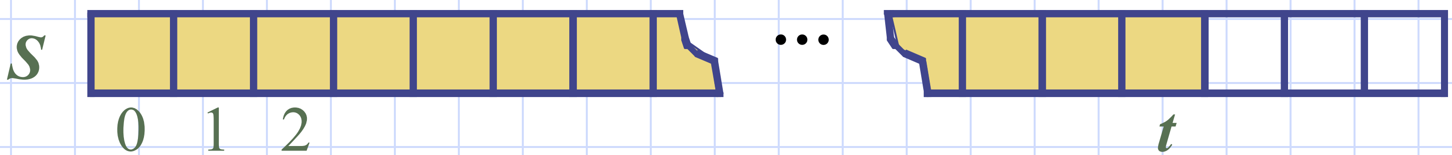


Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable t keeps track of the index of the top element

Algorithm *size*()
  return $t + 1$

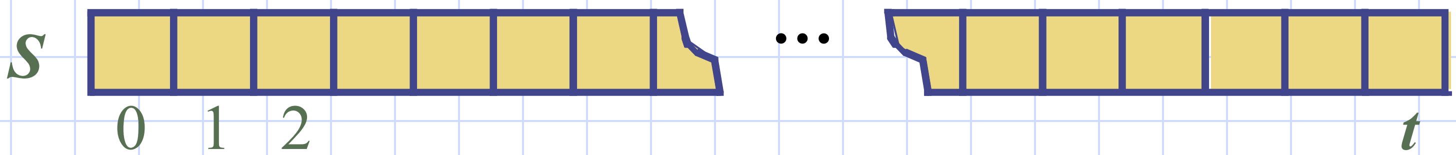
Algorithm *pop*()
  if *isEmpty*() **then**
    throw *EmptyStackException*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a `FullStackException`
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm *push*(*e*)
  if $t = S.length - 1$ **then**
    throw *FullStackException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow e$

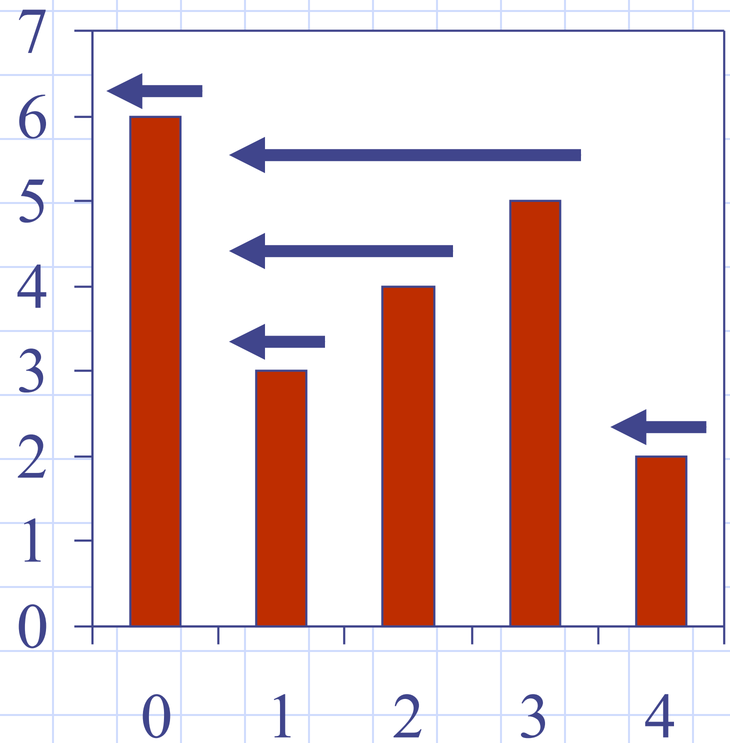


Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Computing Spans

- Using a stack as an auxiliary data structure in an algorithm
- Given an an array X, the span $S[i]$ of $X[i]$ is the maximum number of consecutive elements $X[j]$ immediately preceding $X[i]$ and such that $X[j] \leq X[i]$
- Spans have applications to financial analysis
  - E.g., stock at 52-week high



X	6	3	4	5	2
S	1	1	2	3	1

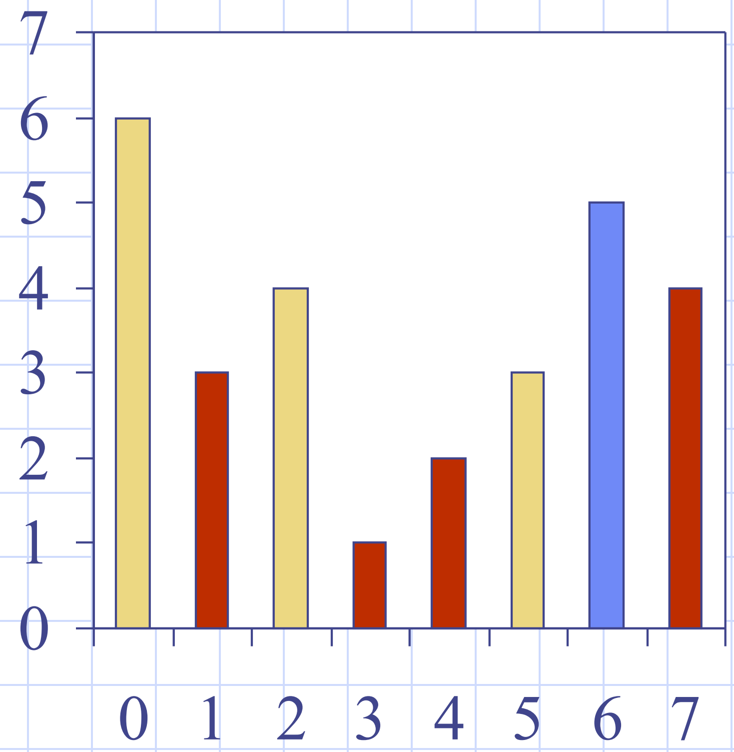
Quadratic Algorithm

Algorithm *spans1*(X, n)	
Input array X of n integers	
Output array S of spans of X	#
$S \leftarrow$ new array of n integers	n
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
$s \leftarrow 1$	n
while $s \leq i$ and $X[i - s] \leq X[i]$	$1 + 2 + \ldots + (n - 1)$
$s \leftarrow s + 1$	$1 + 2 + \ldots + (n - 1)$
$S[i] \leftarrow s$	n
return S	1

- Algorithm ***spans1*** runs in $O(n^2)$ time

Computing Spans with a Stack

- We keep in a stack the indices of the elements visible when "looking back"
- We scan the array from left to right
  - Let i be the current index
  - We pop indices from the stack until we find index j such that $X[i] < X[j]$
  - We set $S[i] \leftarrow i - j$
  - We push x onto the stack



Linear Algorithm

	#
Algorithm *spans2*(X, n)	
$S \leftarrow$ new array of n integers	n
$A \leftarrow$ new empty stack	1
for $i \leftarrow 0$ **to** $n - 1$ **do**	n
while ($\neg A.isEmpty()$ and $X[A.top()] \leq X[i]$) **do**	n
$A.pop()$	n
if $A.isEmpty()$ **then**	n
$S[i] \leftarrow i + 1$	n
else $S[i] \leftarrow i - A.top()$	n
$A.push(i)$	n
return S	1