Introduzione agli Algoritmi

Insertion Sort

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Argomenti della lezione

- Un esempio di Algoritmo
  - Insertion Sort
- Analisi di algoritmi
  - Caso pessimo e caso medio

Problema dell'ordinamento

DEF:

Data una sequenza di numeri n (a_1, a_2, … a_n)

Trovare una permutazione (a'_1, a'_2, … a'_n) della sequenza data tale che $a'_1 < a'_2 < \ldots < a'_n$

Input: (a_1, a_2, … a_n)

Output: (a'_1, a'_2, … a'_n)

Algoritmo: *Insertion Sort*

la posizione i è indicata

30 10 40 20

10 30 40 20

10 30 40 20

10 20 30 40

An Example: Insertion Sort

variabili locali

parametri

passaggio dei parametri

```
InsertionSort(A, n) {
  for i = 2 to n {
      key = A[i]
      j = i - 1;
      while (j > 0) and (A[j] > key) {
            A[j+1] = A[j]
            j = j - 1
      }
      A[j+1] = key
  }
}
```

Analisi di Algoritmi

Analizzare un algoritmo vuol dire determinare le risorse necessarie all'algoritmo.

tempo computazionale

Modello di calcolo

Prima di analizzare un algoritmo abbiamo bisogno di stabilire quale sarà la tecnologia di riferimento quando gli algoritmi saranno realizzati come programmi

Mono-Processore + RAM (Random Access Memory)

assenza di concorrenza e parallelismo

Analisi di Insert-Sort

Il tempo computazionale impiegato dalla procedura Insert-Sort dipende

- *dalla dimensione input*
- dallo ordinamento implicito nella sequenza

- Il tempo di esecuzione viene espresso in funzione della dimensione del problema, cioè dell'input

Dimensione dell'input

Numero degli elementi dell'input

n

la dimensione dell'array per l'ordinamento

Tempo di esecuzione

Numero di operazioni elementari o "passi" eseguiti per il calcolo dell'output

passo ≅ una linea di pseudocodice

Hp:

Ogni passo riferito ad una linea i, è eseguito in un tempo costante c_i

Insertion Sort

Statement	Effort
`InsertionSort(A, n) {`	
`  for i = 2 to n {`	c_1
`    key = A[i]`	c_2
`    j = i - 1;`	c_3
`    while (j > 0) and (A[j] > key) {`	c_4
`      A[j+1] = A[j]`	c_5
`      j = j - 1`	c_6
`    }`	0
`    A[j+1] = key`	c_7
`  }`	0
`}`	

Calcolo del Tempo d'esecuzione

Indichiamo con n_i il numero di volte che un passo i viene eseguito

$$T(n) = c_1 n_1 + c_2 n_2 + \ldots c_n n_n = \Sigma_{i=1}^{n} c_i n_i$$

Analisi del Caso Peggiore e Caso Medio

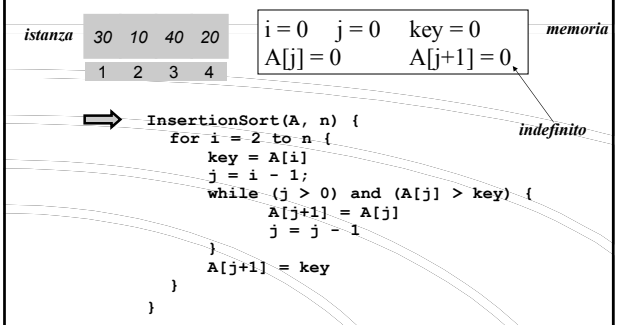
Il tempo d'esecuzione nel caso peggiore di un algoritmo rappresenta un limite superiore (upper-bound) sui tempi d'esecuzione per qualsiasi input.

La sua conoscenza garantisce che l'algoritmo non impiegherà mai tempi maggiori

Caso medio è approssimativamente negativo quanto il caso peggiore.

Come determinare il caso medio?

An Example: Insertion Sort

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  for i = 2 to n {
      key = A[i]
      j = i - 1;
      while (j > 0) and (A[j] > key) {
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      }
      A[j+1] = key
  }
}
```

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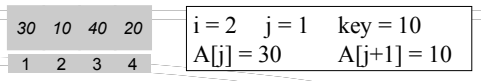


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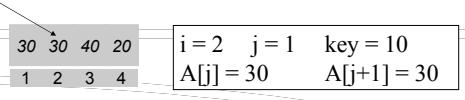


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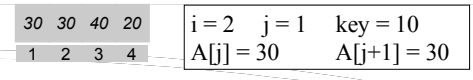


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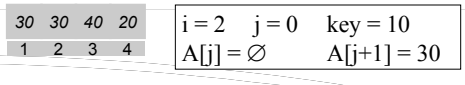


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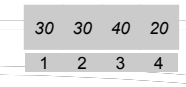

i = 2 j = 0 key = 10
A[j] = ∅ A[j+1] = 30

```
InsertionSort(A, n) {
  for i = 2 to n {
      key = A[i]
      j = i - 1;
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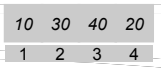

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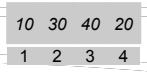

i = 3 j = 0 key = 10
A[j] = ∅ A[j+1] = 10

```
InsertionSort(A, n) {
  for i = 2 to n {
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      }
      A[j+1] = key
  }
}
```

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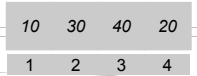

i = 3 j = 0 key = 40
A[j] = ∅ A[j+1] = 10

```
InsertionSort(A, n) {
  for i = 2 to n {
      key = A[i]
      j = i - 1;
      while (j > 0) and (A[j] > key) {
            A[j+1] = A[j]
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      }
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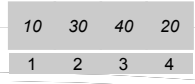

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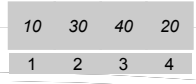

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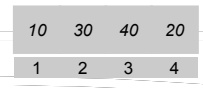

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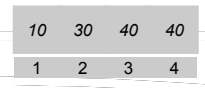

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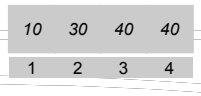

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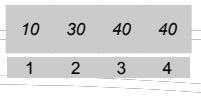

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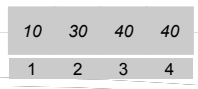

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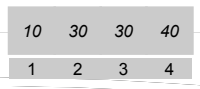

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}
```

An Example: Insertion Sort

10	30	30	40
1	2	3	4

i = 4 j = 1 key = 20
A[j] = 10 A[j+1] = 30

```
InsertionSort(A, n) {
  for i = 2 to n {
      key = A[i]
      j = i - 1;
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  }
}
```

Done!

Animating Insertion Sort

- Check out the Animator, a java applet at:
- Try it out with random, ascending, and descending inputs

Insertion Sort

What is the precondition for this loop?

```
InsertionSort(A, n) {
   for i = 2 to n {
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       j = i - 1;
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             j = j - 1
       }
       A[j+1] = key
   }
}
```